Consider the expression $(13x^7 - 17x^3)^{27}$. **SCORE: _____ / 25 PTS**

[a] Write the expansion of the expression using sigma notation. **Your answer may use ! but not $_nC_r$ notation.** Simplify all exponents.

$$\sum_{r=0}^{27} {}_{27}C_r (13x^7)^{27-r}(-17x^3)^r = \sum_{r=0}^{27} \frac{27!}{r!(27-r)!} 13^{27-r}(-17)^r x^{7(27-r)+3r}$$

$$= \sum_{r=0}^{27} \frac{27!}{r!(27-r)!} 13^{27-r}(-17)^r x^{189-4r}$$

[b] Find the coefficient of x^{129} in the expansion. **Your answer may use ! but not $_nC_r$ notation.**

$189-4r=129$

$-4r=-60$

$r=15$

$$\frac{27!}{15!\,12!} 13^{12}(-17)^{15} = -\frac{27!}{15!\,12!} 13^{12} 17^{15}$$

Simplify $\dfrac{(7n-4)!}{(7n-1)!}$. **SCORE: _____ / 10 PTS**

$$\frac{\cancel{(7n-4)!}}{(7n-1)(7n-2)(7n-3)\cancel{(7n-4)!}} = \frac{1}{(7n-1)(7n-2)(7n-3)}$$

Eliminate the parameter to find rectangular equations corresponding to the parametric equations $\begin{array}{l} x = \dfrac{t}{2-t} \\ y = \dfrac{t+1}{t-3} \end{array}$. **SCORE: _____ / 15 PTS**

For your final answer, write y as a simplified function of x.

$x(2-t)=t$

$2x-xt=t$

$2x=t+xt$

$2x=t(1+x)$

$t=\dfrac{2x}{1+x}$

$$y=\frac{t+1}{t-3} = \frac{\frac{2x}{1+x}+1}{\frac{2x}{1+x}-3} = \frac{2x+(1+x)}{2x-3(1+x)} = \frac{3x+1}{-x-3} = -\frac{3x+1}{x+3}$$

GJ is standing 21 feet from HJ, who is 6 feet tall. GJ throws a football at 30 feet per second in HJ's direction, at an angle of 67.38° with the horizontal, from an initial height of 5 feet. **SCORE: _____ / 25 PTS**

NOTE: $\sin 67.38^\circ = \frac{12}{13}$ **and** $\cos 67.38^\circ = \frac{5}{13}$

[a] Write parametric equations for the position of the football.

$x = (30\cos 67.38^\circ)t = \frac{150}{13}t$

$y = 5 + (30\sin 67.38^\circ)t - 16t^2 = 5 + \frac{360}{13}t - 16t^2$

[b] Does the football hit HJ, go over HJ's head, or hit the ground before reaching HJ ?

$x = 21$

$\frac{150}{13}t = 21$

$t = 1.82$

$y = 5 + \frac{360}{13}(1.82) - 16(1.82)^2$

$y = 2.4016$

$0 < 2.4016 < 6$

THE FOOTBALL HITS HJ

IJ spent 13 hours studying the first week of the quarter. Each week afterwards, IJ's study time was 9% more than the previous week. If the quarter was 12 weeks long, how much time did IJ study over the entire quarter ? **SCORE: _____ / 10 PTS**

$13 + 13(1.09) + 13(1.09)^2 + \cdots + 13(1.09)^{11}$ GEOMETRIC, $r = 1.09$

$= \dfrac{13(1.09^{12} - 1)}{1.09 - 1} = 261.83$ HOURS

Find the sum of the series $249 + 241 + 233 + 225 + 217 + \cdots - 375$. **SCORE: _____ / 15 PTS**

ARITHMETIC, $d = -8$

$249 - 8(n-1) = -375$

$-8(n-1) = -624$

$n - 1 = 78$

$n = 79$

$S_{79} = \frac{1}{2}(79)(249 + -375) = -4977$

Using mathematical induction, prove that $\sum_{i=1}^{n} 4^{n+1} = \frac{4^{n+2}-16}{3}$ for all positive integers n. **SCORE: _____ / 25 PTS**

Do NOT use the finite geometric series formula in your proof.

BASIS STEP: WHEN $n=1$, $\sum_{i=1}^{1} 4^{i+1} = 4^2 = 16$

$$\frac{4^3-16}{3} = \frac{48}{3} = 16$$

INDUCTIVE STEP:

ASSUME $\sum_{i=1}^{k} 4^{i+1} = \frac{4^{k+2}-16}{3}$ FOR SOME ARBITRARY BUT PARTICULAR INTEGER k

$$\sum_{i=1}^{k+1} 4^{i+1} = \sum_{i=1}^{k} 4^{i+1} + 4^{k+2}$$

$$= \frac{4^{k+2}-16}{3} + 4^{k+2}$$

$$= \frac{4^{k+2}-16+3\cdot 4^{k+2}}{3}$$

$$= \frac{4\cdot 4^{k+2}-16}{3}$$

$$= \frac{4^{k+3}-16}{3}$$

$$= \frac{4^{(k+1)+2}-16}{3}$$

BY MI, $\sum_{i=1}^{n} 4^{i+1} = \frac{4^{n+2}-16}{3}$ FOR ALL POSITIVE INTEGERS n

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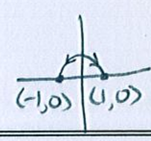
Describe the difference between the curves with parametric equations $\begin{matrix} x=\sin t \\ y=\cos^2 t \end{matrix}$ and $\begin{matrix} x=-t^2 \\ y=1-t^4 \end{matrix}$. **SCORE: _____ / 10 PTS**

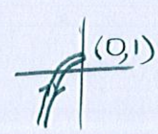
Discuss the rectangular equation(s) of the graphs, as well as the orientation and portion of the graph corresponding to the parametric equations.

BOTH CURVES CORRESPOND TO $y=1-x^2$ 

AS t GOES FROM $-\infty$ TO ∞

$x=\sin t$ OSCILLATES BETWEEN -1 AND 1 BUT $x=-t^2$ GOES FROM $-\infty$ TO 0 TO $-\infty$





Use sigma notation to write the series $\frac{32}{6} - \frac{80}{24} + \frac{200}{120} - \frac{500}{720} + \cdots - \frac{3125}{40320}$. ← GEOMETRIC, $r=-2.5$ **SCORE: _____ / 15 PTS**

$\times 4 \quad \times 5 \quad \times 6$

$3! \quad 4! \quad 5! \quad 6! \quad \quad 8!$

$$\sum_{n=1}^{6} \frac{32(-2.5)^{n-1}}{(n+2)!} \quad \text{OR} \quad \sum_{n=3}^{8} \frac{32(-2.5)^{n-3}}{n!} \quad \text{OR} \quad \sum_{n=0}^{5} \frac{32(-2.5)^{n}}{(n+3)!}$$